

Stable Constant Mean Curvature Hypersurfaces

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Abstract

Let \mathcal{N}^{n+1} be a Riemannian manifold with sectional curvatures uniformly bounded from below. When $n = 3, 4$, we prove that there are no complete (strongly) stable H -hypersurfaces, without boundary, provided $|H|$ is large enough. In particular we prove that there are no complete strongly stable H -hypersurfaces in \mathbb{R}^{n+1} without boundary, $H \neq 0$.

1 Introduction

Consider a Riemannian manifold \mathcal{N} of dimension $n + 1$ with sectional curvatures uniformly bounded from below; denote by $\sec(\mathcal{N})$ the infimum of the sectional curvatures of \mathcal{N} . Let M be an immersed submanifold of codimension one and let H be the mean curvature of M in the metric induced by the immersion. If H is constant, we call M an H -hypersurface. We prove the following diameter estimate.

Theorem 1 *Let $M^n \subset \mathcal{N}^{n+1}$ be a stable complete H -submanifold, $n = 3, 4$. There exists a constant $c = c(n, H, \sec(\mathcal{N}))$ such that for any $p \in M$ one has: $\text{dist}_M(p, \partial M) \leq c$ whenever $|H| > 2\sqrt{|\min\{0, \sec(\mathcal{N})\}|}$.*

For the definition of stability see Section 2. Particular cases of the previous Theorem in \mathbb{R}^3 , \mathbb{H}^3 , $\mathbb{H}^2 \times \mathbb{R}$ and any homogeneously regular three manifold are proved in [8], [4], [6], [7] respectively.

We wonder if Theorem 1 holds in all dimensions.

Corollary 1 *Let M^n be a complete stable H -hypersurface of \mathcal{N}^{n+1} . If $n = 3, 4$ and $|H| > 2\sqrt{|\min\{0, \sec(\mathcal{N})\}|}$, then $\partial M \neq \emptyset$.*

In [11] it is proved that an H -hypersurface in \mathbb{R}^{n+1} , with finite total curvature, is minimal, so, if it is stable, it is a hyperplane (cf. [3]). For $n = 3, 4$, we are able to generalize this result in the following sense. We do not need the finite total curvature hypothesis on M and the ambient space can be any manifold with uniformly bounded sectional curvature, provided the mean curvature $|H|$ is large enough (See Corollary 1).

As a consequence of the diameter estimate in Theorem 1, we have the Maximum Principle at Infinity.

Theorem 2 *Let \mathcal{N}^{n+1} have uniformly bounded sectional curvature, $n = 3, 4$. If $|H| > 2\sqrt{|\min\{0, \sec(\mathcal{N})\}|}$ and M_1, M_2 are properly embedded H -hypersurfaces in \mathcal{N}^{n+1} , which*

bound a connected domain W , then the mean curvature vector points out of W along the boundary of W .

The proof of Theorem 2 is the same as in [7], where the result is proved for $n = 2$.

2 Proofs

Let M be a H -hypersurface in a manifold \mathcal{N} and let N be a unit vector field normal to M in \mathcal{N} . The stability operator of M is $L = \Delta + |A|^2 + Ric(N)$ where $Ric(N)$ is the Ricci curvature of the ambient manifold \mathcal{N} in the direction of N and A is the shape operator of the immersion. We say that M is *stable* if

$$-\int_M uLu \geq 0,$$

for any smooth function u with compact support on M . Our definition of stability is usually known as *strong stability*. The usual definition of stability (*weak stability*) also requires the test function u to satisfy $\int_M u = 0$. Geodesic spheres in a space form are weakly stable but they are not stable (cf. [2]). We remark that the solutions of the Plateau problem are stable hypersurfaces in our sense as well as any H -hypersurface transverse to some Killing vector field of the ambient manifold. The proof of the latter is standard (cf. for example [6]). For further relations between the two notions of stability see [1] and [2].

Proof of Theorem 1. Consider the traceless operator $\Phi = A - HI$. One can write the stability operator of M in terms of Φ , namely, $L = \Delta + |\Phi|^2 + nH^2 + Ric(N)$. Since M is stable, there exists a function $u > 0$ on M such that $Lu = 0$ on M (cf. [5]).

Denote by ds^2 the metric on M induced by the immersion in \mathcal{N} and let $d\tilde{s}^2 = u^{2k}ds^2$, with $\frac{5(n-1)}{4n} \leq k < \frac{4}{n-1}$. This choice of k will be justified later. Notice that, in order to have some k satisfying the previous inequality, one needs $n = 3, 4$.

Consider $p \in M$ and let $r > 0$ be such that the intrinsic ball B_r of M , centered at p of ds -radius r , is contained in the interior of M . Let γ be a $d\tilde{s}$ -minimizing geodesic in B_r joining p to ∂B_r . Let a be the ds -length of γ . Then $a \geq r$ and it is enough to prove that there exists a constant $c(n, H, sec(\mathcal{N}))$ such that $a \leq c$.

Let R and \tilde{R} be the curvature tensor of M in the metric ds and $d\tilde{s}$, respectively. Choose a basis $\{\tilde{e}_1 = \frac{\partial \gamma}{\partial \tilde{s}}, \tilde{e}_2, \dots, \tilde{e}_n\}$ orthonormal for the metric $d\tilde{s}$, such that $\tilde{e}_2, \dots, \tilde{e}_n$ are parallel along γ and let $\tilde{e}_{n+1} = N$. The basis $\{e_1 = \frac{\partial \gamma}{\partial s} = u^k \tilde{e}_1, e_2 = u^k \tilde{e}_2, \dots, e_n = u^k \tilde{e}_n\}$ is orthonormal for the metric ds . Denote by R_{11} and \tilde{R}_{11} the Ricci curvature in the direction of e_1 for the metric ds and $d\tilde{s}$ respectively. Let \hat{R} be the curvature tensor of the ambient manifold \mathcal{N} and write $Ric(N) = \hat{R}_{n+1, n+1}$.

Let \tilde{r} be the length of γ in the $d\tilde{s}$ metric. Since γ is $d\tilde{s}$ minimizing, by the second variation formula, one has

$$\int_0^{\tilde{r}} \left[(n-1) \left(\frac{d\varphi}{d\tilde{s}} \right)^2 - \tilde{R}_{11} \varphi^2 \right] d\tilde{s} \geq 0, \quad (1)$$

for any smooth function φ such that $\varphi(0) = \varphi(\tilde{r}) = 0$.

As it is proved in the Appendix

$$\tilde{R}_{11} = u^{-2k} \left\{ R_{11} - k(n-2)(\ln u)_{ss} - k \frac{\Delta u}{u} + k \frac{|\nabla u|^2}{u^2} \right\}. \quad (2)$$

Now use that $Lu = (\Delta + |\Phi|^2 + nH^2 + \widehat{R}_{n+1,n+1})u = 0$ to obtain

$$\tilde{R}_{11} = u^{-2k} \left\{ R_{11} - k(n-2)(\ln u)_{ss} + k(|\Phi|^2 + nH^2 + \widehat{R}_{n+1,n+1}) + k \frac{|\nabla u|^2}{u^2} \right\}. \quad (3)$$

From the Gauss equation one has

$$R_{ijij} = \widehat{R}_{ijij} + h_{ii}h_{jj} - h_{ij}^2, \quad (4)$$

which can be rewritten as

$$R_{ijij} = \widehat{R}_{ijij} + (\Phi_{ii} + H)(\Phi_{jj} + H) - (\Phi_{ij} + H\delta_{ij})^2.$$

Taking $i = 1$ and summing up in $j = 2, \dots, n$ we obtain

$$R_{11} = \sum_{j=2}^n \widehat{R}_{1j1j} + \sum_{j=2}^n \Phi_{11}\Phi_{jj} + (n-2)H\Phi_{11} + \sum_{j=1}^n \Phi_{jj}H + (n-1)H^2 - \sum_{j=2}^n \Phi_{1j}^2.$$

Since $\sum_{j=1}^n \Phi_{jj} = 0$, we have

$$R_{11} = \sum_{j=2}^n \widehat{R}_{1j1j} - \Phi_{11}^2 + (n-2)H\Phi_{11} + (n-1)H^2 - \sum_{j=2}^n \Phi_{1j}^2.$$

Replacing the last relation in equation (3), yields

$$\begin{aligned} \tilde{R}_{11} &= u^{-2k} \left[\sum_{j=2}^n \widehat{R}_{1j1j} + k\widehat{R}_{n+1,n+1} + (kn+n-1)H^2 + (n-2)H\Phi_{11} \right] \\ &\quad + u^{-2k} \left[k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2 - k(n-2)(\ln u)_{ss} + k \frac{|\nabla u|^2}{u^2} \right]. \end{aligned}$$

Combining the last equation with inequality (1) gives (by abuse of notation we denote again by φ the composition $\varphi \circ \tilde{s}$, hence $\varphi(0) = \varphi(a) = 0$)

$$\begin{aligned} (n-1) \int_0^a (\varphi_s)^2 u^{-k} ds &\geq \int_0^a \varphi^2 u^{-k} \left[\sum_{j=2}^n \widehat{R}_{1j1j} + k\widehat{R}_{n+1,n+1} \right] ds \\ &+ \int_0^a \varphi^2 u^{-k} \left[(kn+n-1)H^2 + (n-2)H\Phi_{11} + k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2 \right] ds \\ &\quad - \int_0^a \varphi^2 u^{-k} \left[k(n-2)(\ln u)_{ss} + k \frac{|\nabla u|^2}{u^2} \right] ds. \end{aligned}$$

Replace φ by $\varphi u^{\frac{k}{2}}$ to get rid of u^k in the denominator. The last relation becomes

$$\begin{aligned}
(n-1) \int_0^a (\varphi_s)^2 ds + k(n-1) \int_0^a \varphi \varphi_s u_s u^{-1} ds + \frac{k^2(n-1)}{4} \int_0^a \varphi^2 u_s^2 u^{-2} ds \\
\geq \int_0^a \varphi^2 \left[\sum_{j=2}^n \widehat{R}_{1j1j} + k \widehat{R}_{n+1,n+1} \right] ds \\
+ \int_0^a \varphi^2 \left[(kn+n-1)H^2 + (n-2)H\Phi_{11} + k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2 \right] ds \\
- \int_0^a \varphi^2 \left[k(n-2)(\ln u)_{ss} + k \frac{|\nabla u|^2}{u^2} \right] ds. \tag{5}
\end{aligned}$$

Integration by parts gives

$$\int \varphi^2 (\ln u)_{ss} ds = -2 \int \varphi \varphi_s \frac{u_s}{u} ds.$$

Then, replacing in inequality (5), we obtain

$$\begin{aligned}
(n-1) \int_0^a (\varphi_s)^2 ds \geq k(n-3) \int_0^a \varphi \varphi_s u_s u^{-1} ds - \frac{(n-1)}{4} \int_0^a \varphi^2 (\ln u^k)_s^2 ds \\
+ k \int_0^a \varphi^2 \frac{|\nabla u|^2}{u^2} ds + \int_0^a \varphi^2 \left[k \widehat{R}_{n+1,n+1} + \sum_{j=2}^n \widehat{R}_{1j1j} \right] ds \\
+ \int_0^a \varphi^2 \left[(kn+n-1)H^2 + (n-2)H\Phi_{11} + k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2 \right] ds,
\end{aligned}$$

that is

$$\begin{aligned}
(n-1) \int_0^a (\varphi_s)^2 ds \geq k(n-3) \int_0^a \varphi \varphi_s u_s u^{-1} ds + \left[\frac{1}{k} - \frac{(n-1)}{4} \right] \int_0^a \varphi^2 (\ln u^k)_s^2 ds \\
+ \int_0^a \varphi^2 \left[k \widehat{R}_{n+1,n+1} + \sum_{j=2}^n \widehat{R}_{1j1j} \right] ds \\
+ \int_0^a \varphi^2 \left[(kn+n-1)H^2 + (n-2)H\Phi_{11} + k|\Phi|^2 - \Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2 \right] ds. \tag{6}
\end{aligned}$$

We now use that $a^2 + b^2 \geq -2ab$ with $a = (n-2)H$ and $b = \frac{\Phi_{11}}{2}$, to obtain

$$(n-2)^2 H^2 + \frac{\Phi_{11}^2}{4} \geq -(n-2)H\Phi_{11}.$$

Replacing in inequality (6) yields

$$\begin{aligned}
(n-1) \int_0^a (\varphi_s)^2 ds &\geq k(n-3) \int_0^a \varphi \varphi_s u_s u^{-1} ds + \left[\frac{1}{k} - \frac{(n-1)}{4} \right] \int_0^a \varphi^2 (\ln u^k)_s^2 ds \\
&\quad + \int_0^a \varphi^2 \left[k \widehat{R}_{n+1, n+1} + \sum_{j=2}^n \widehat{R}_{1j1j} + (kn - n^2 + 5n - 5)H^2 + \right] ds \\
&\quad + \int_0^a \left[k|\Phi|^2 - \frac{5}{4}\Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2 \right] ds. \tag{7}
\end{aligned}$$

We will now prove that the last term in inequality (7) is greater or equal than zero. We know that

$$|\Phi|^2 \geq \Phi_{11}^2 + \Phi_{22}^2 + \cdots + \Phi_{nn}^2 + 2 \sum_{j=2}^n \Phi_{1j}^2$$

and since $\sum_{j=1}^n \Phi_{jj} = 0$, we have

$$|\Phi|^2 \geq \frac{n}{n-1} \Phi_{11}^2 + 2 \sum_{j=2}^n \Phi_{1j}^2. \tag{8}$$

Since $k \geq \frac{5(n-1)}{4n}$, using inequality (8), we obtain

$$\begin{aligned}
&k|\Phi|^2 - \frac{5}{4}\Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2 \\
&\geq \frac{5}{4}\Phi_{11}^2 + \frac{5(n-1)}{2n} \sum_{j=2}^n \Phi_{1j}^2 - \frac{5}{4}\Phi_{11}^2 - \sum_{j=2}^n \Phi_{1j}^2 = \frac{3n-5}{2n} \sum_{j=2}^n \Phi_{1j}^2 \geq 0.
\end{aligned}$$

Then, inequality (7) yields

$$\begin{aligned}
(n-1) \int_0^a (\varphi_s)^2 ds &\geq (n-3) \int_0^a \varphi \varphi_s (\ln u^k)_s ds + \left[\frac{1}{k} - \frac{(n-1)}{4} \right] \int_0^a \varphi^2 (\ln u^k)_s^2 ds \\
&\quad + \int_0^a \varphi^2 \left[k \widehat{R}_{n+1, n+1} + \sum_{j=2}^n \widehat{R}_{1j1j} + (kn - n^2 + 5n - 5)H^2 \right] ds. \tag{9}
\end{aligned}$$

We now use that $a^2 + b^2 \geq -2ab$ with $a = \left(\frac{1}{k} - \frac{(n-1)}{4} \right)^{\frac{1}{2}} \varphi (\ln u^k)_s$ and $b = \frac{(n-3)}{2} \left(\frac{1}{k} - \frac{(n-1)}{4} \right)^{-\frac{1}{2}} \varphi_s$, to obtain

$$\left(\frac{1}{k} - \frac{(n-1)}{4} \right) \varphi^2 (\ln u^k)_s^2 + \frac{(n-3)^2}{4} \left(\frac{1}{k} - \frac{(n-1)}{4} \right)^{-1} \varphi_s^2 \geq -(n-3) \varphi \varphi_s (\ln u^k)_s.$$

The last inequality together with inequality (9) gives

$$(n-1) \int_0^a (\varphi_s)^2 ds \geq -\frac{(n-3)^2}{4} \left(\frac{1}{k} - \frac{(n-1)}{4} \right)^{-1} \int_0^a (\varphi_s)^2 ds + \int_0^a \varphi^2 \left[(kn - n^2 + 5n - 5)H^2 + k\widehat{R}_{n+1,n+1} + \sum_{j=2}^n \widehat{R}_{1j1j} \right] ds.$$

Then setting $A = \frac{4[k(2-n)+(n-1)]}{4-k(n-1)}$ and making a suitable choice of a positive constant B , we can rewrite the last inequality as

$$A \int_0^a (\varphi_s)^2 ds \geq B \int_0^a \varphi^2 ds. \quad (10)$$

We remark that A is positive as soon as $k < \frac{4}{n-1} \leq \frac{n-1}{n-2}$. We now want to choose B such that

$$0 < B \leq (kn - n^2 + 5n - 5)H^2 + \left(k\widehat{R}_{n+1,n+1} + \sum_{j=2}^n \widehat{R}_{1j1j} \right).$$

When the curvature of the ambient manifold is non-negative, we set $B = (kn - n^2 + 5n - 5)H^2$, which is positive if $H \neq 0$ (remember that $k > \frac{5(n-1)}{4n}$ and that $n = 3, 4$). In this case we can set $c_1 = 0$.

Otherwise, we proceed as follows. By a straightforward computation one has

$$k\widehat{R}_{n+1,n+1} + \sum_{j=2}^n \widehat{R}_{1j1j} \geq (kn + n - 1) \inf\{\text{sectional curvatures of } \mathcal{N}\} = \text{sec}(\mathcal{N}),$$

then we set $B = (kn - n^2 + 5n - 5)H^2 + (kn + n - 1)\text{sec}(\mathcal{N})$. If

$$H^2 > \frac{kn + n - 1}{kn - n^2 + 5n - 5} \text{sec}(\mathcal{N}), \quad (11)$$

then B is positive. In this case, one can set $c_1 = 2\sqrt{|\text{sec}(\mathcal{N})|}$ (using the restrictions on k one can prove that $\frac{kn+n-1}{kn-n^2+5n-5} < 4$).

Integration by parts in inequality (10) yields

$$\int_0^a (\varphi_{ss}A + B\varphi)\varphi ds \leq 0.$$

Choosing $\varphi = \sin(\pi sa^{-1})$, $s \in [0, a]$ one has

$$\int_0^a \left[B - \frac{A\pi^2}{a^2} \right] \sin^2(\pi sa^{-1}) ds \leq 0.$$

Finally

$$B - \frac{A\pi^2}{a^2} \leq 0,$$

and this gives the desired inequality, if we choose

$$c = \frac{2\pi\sqrt{k(2-n) + (n-1)}}{\sqrt{(4-k(n-1))[(kn-n^2+5n-5)H^2 + (kn+n-1)\min\{0, \sec(\mathcal{N})\}]}}.$$

□

Proof of Corollary 1. Assume that such an M exists. In the proof of Theorem 1, we showed that the radius of an intrinsic disc of M , that does not touch ∂M , is at most c . Hence, when $\partial M = \emptyset$, the diameter of M is at most c and then M is compact. As M is stable, there exists a positive function f on M such that $L(f) = 0$ (cf. [5]). Let $p \in M$ be a minimum of the function f . At p , one has:

$$0 \leq \Delta f(p) = -(|\Phi|^2(p) + nH^2 + \widehat{R}_{n+1, n+1}(p))f(p).$$

By our choice of H , the potential $|\Phi|^2 + nH^2 + \widehat{R}_{n+1, n+1}$ is strictly positive on M , hence the previous inequality yields a contradiction.

□

3 Appendix

The transformation law of the curvature under the conformal change of the metric $d\tilde{s}^2 = u^{2k} ds^2$ is the following (cf. [9] page 184 and [10] formula (4))

$$\begin{aligned} \widetilde{R}_{11} &= \widetilde{Ric}\left(\frac{\partial\gamma}{\partial\tilde{s}}, \frac{\partial\gamma}{\partial\tilde{s}}\right) = \left\{ Ric\left(\frac{\partial\gamma}{\partial\tilde{s}}, \frac{\partial\gamma}{\partial\tilde{s}}\right) - k(n-2)Hess(\ln u) \left(\frac{\partial\gamma}{\partial\tilde{s}}, \frac{\partial\gamma}{\partial\tilde{s}}\right) \right. \\ &\quad \left. + k^2(n-2)\left|\frac{\partial\gamma}{\partial\tilde{s}}(\ln u)\right|^2 - [k\Delta(\ln u) + k^2(n-2)|\nabla \ln u|^2]u^{-2k} \right\}. \end{aligned} \quad (12)$$

In order to simplify this equation we need to compute $\nabla_{\frac{\partial\gamma}{\partial\tilde{s}}}\frac{\partial\gamma}{\partial\tilde{s}}$.

Using the relation between the connections of conformal metrics we obtain

$$\widetilde{\nabla}_{\frac{\partial\gamma}{\partial\tilde{s}}}\frac{\partial\gamma}{\partial\tilde{s}} = \nabla_{\frac{\partial\gamma}{\partial\tilde{s}}}\frac{\partial\gamma}{\partial\tilde{s}} + 2k \langle \nabla \ln u, \frac{\partial\gamma}{\partial\tilde{s}} \rangle \frac{\partial\gamma}{\partial\tilde{s}} - k \nabla \ln u.$$

Since γ is geodesic in the $d\tilde{s}^2$ metric we have that $\widetilde{\nabla}_{\frac{\partial\gamma}{\partial\tilde{s}}}\frac{\partial\gamma}{\partial\tilde{s}} = 0$ and thus

$$\widetilde{\nabla}_{\frac{\partial\gamma}{\partial\tilde{s}}}\frac{\partial\gamma}{\partial\tilde{s}} = k \langle \nabla \ln u, \frac{\partial\gamma}{\partial\tilde{s}} \rangle \frac{\partial\gamma}{\partial\tilde{s}}.$$

The last two equations yield

$$\nabla_{\frac{\partial\gamma}{\partial\tilde{s}}}\frac{\partial\gamma}{\partial\tilde{s}} = k(\nabla \ln u)^\perp, \quad (13)$$

where $(\nabla \ln u)^\perp$ means the component of $\nabla \ln u$ perpendicular to $\frac{\partial\gamma}{\partial\tilde{s}}$. Now we observe that

$$\begin{aligned}
\text{Hess}(\ln u)\left(\frac{\partial\gamma}{\partial\bar{s}}, \frac{\partial\gamma}{\partial\bar{s}}\right) &= u^{-2k}\left((\ln u)_{ss} - \left(\nabla_{\frac{\partial\gamma}{\partial\bar{s}}}\frac{\partial\gamma}{\partial\bar{s}}\right)\ln u\right) \\
&= u^{-2k}\left((\ln u)_{ss} - k|(\nabla\ln u)^\perp|^2\right),
\end{aligned}$$

where in the last equality we use (13). Replacing this last equation in (12) one obtains

$$\begin{aligned}
\tilde{R}_{11} &= u^{-2k}\left\{R_{11} - k(n-2)(\ln u)_{ss} + k^2(n-2)|(\nabla\ln u)^\perp|^2\right. \\
&\quad \left.+ k^2(n-2)|(\ln u)_s|^2 - [k\Delta(\ln u) + k^2(n-2)|\nabla\ln u|^2]\right\},
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
\tilde{R}_{11} &= u^{-2k}\{R_{11} - k(n-2)(\ln u)_{ss} - k\Delta(\ln u)\} \\
&= u^{-2k}\left\{R_{11} - k(n-2)(\ln u)_{ss} - k\frac{\Delta u}{u} + k\frac{|\nabla u|^2}{u^2}\right\}.
\end{aligned} \tag{14}$$

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